On the Ontology and Epistemology of Musical Intervals … or Bye Bye Pythagoras.

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1: A practitioner's story.

We begin this reflection with accounts of two specific, practical, personal experiences which deal with the venerable topic of musical intervals from two quite different perspectives. Nevertheless, despite their anecdotal nature, these two stories sum up situations which have led us to reflect on a broader, more general, aspect of the topic of musical intervals: viz the *ontology* of musical intervals (*what* exactly it is that we consider we know them to be) and about the *epistemology* of musical intervals (*how* we know *what* we know). We shall then attempt to relate such specific, practical and personal experiences to research by practising musicians and by scholars in various branches of musicology, and indeed to research from the perspective of other disciplines, which have something illuminating to say about the nature of musical intervals. Naturally, our attempt to illuminate such a fundamental topic must needs refer to historical attempts by many generations of scholars, scientists and practitioners, extending back — if "extending back" means as far back as Pythagoras (circa 569 – circa 475 BC) — over two and a half millennia.

We can approach this question from two (at least) different angles: through logical argument and through empirical observation of how we experience intervals in the course of our practice of music. Nevertheless, both of these angles are deeply influenced by our *weltanschauung*: how we see the world generally from the perspective of our own situation in time and space. In the concluding sections of this paper we shall endeavour to take this fact into account.

2: Observations from the perspective of a specific compositional and performance project …

Our first account comes from the perspective of composition and performance. It comprises a series of observations made in the course of composing and performing a series of new vocal and instrumental compositions by our first author (GH), and it summarises the music-theoretical fallout from an overview of that experience. The compositions in question are a series of six duets for two singers and digital harmonium. The choice of the digital harmonium as the accompanying instrument was made partly just because, like all keyboard instruments, the tuning of every pitch is fixed precisely in advance — computer-controlled in fact in this case — but especially because the scale used in three of the duets was the 19-EDO scale (the scale with nineteen equal divisions of the octave), which is not generally available on more conventional instruments, while, in the other

three songs, the conventional, ubiquitous 12-EDO scale (ie the scale with twelve equal divisions of the octave) was used, and the digital harmonium makes switching between the two scales trivially easy (which is clearly not true of conventional keyboards). On the other hand, when we talk about *tuning* with respect to the two vocal melodies, we would probably be more accurate to speak of *intonation*, because every note singers sing is determined by moment-to-moment choices and decisions of the performers, and probably involuntary physical reactions, including such phenomena as "muscle-memory", and indeed their intonation varies not only by the note, but over the course of the duration of the note as well. Moreover, as every singer knows, the pitch of every note — onset, middle portion and termination — is influenced by the pitch, rhythm and other characteristics of the notes of her melodic line which precede and follow, and by the notes which are being sounded simultaneously or in proximity by other instruments and voices, and perhaps by other considerations as well. Such moment-to-moment control of intonation is characteristic of stringed, woodwind and brass instruments too, of course, but singers provide the *locus classicus*.

To return to the digital harmonium for a moment, there were also cultural reasons for the choice, as well as the harmonium's precise 19-EDO tuning, namely an element of "world music" sensibility, and particularly of *Qawwali* music. *Qawwali* is the sufi genre which features the harmonium — an instrument of European origin which musicians of the South Asian sub-continent have made completely their own in recent generations, albeit in a portable form uniquely of the sub-continent. But as far as this particular project goes, in the pieces from which discussion proceeds here, the juxtaposition of voices and digital harmonium also serves the particular purpose of exploring empirically aspects of the ontology and epistemology of melodic and harmonic intervals from a twenty-first century practical perspective.

The six duets are from a cycle of *Sufi Fragments*, to short texts (couplets extracted from longer, multi-couplet *ghazals* or *ghazal*-like poems) by the medieval Turkish Sufi folk poet Yunus Emre (1238-1320) in English translations by Süha Faiz, a Turkish Cypriot, born in Cyprus in the days of the British administration there, but who was sent to England for his education as a youth, and subsequently spent most of his life as a civil servant in the (then) British Foreign and Commonwealth Office (Faiz 2007). The reasons for choosing to start with a consideration of excerpts from this song-cycle were, as mentioned above, the use of both 12-EDO and 19-EDO scales, but also that each song is very short: in fact, just one minute long (restricting the material to be considered to manageable proportions). Nevertheless, despite the different scales, the two sets of duets share some chord-structures which any musician can easily recognise as perceptually similar, and the characteristic idiom of the two sets of duets is not substantially different. For example (to take a random instance): the harmony consisting of the pitchclasses GCDF, sometimes known in contemporary popular, folk-derived, "roots", jazz and world music (hereinafter "PFRJW") circles as the "Major-minor 7th with suspended 4th" chord, is used in both sets of songs.

The harmonic character of *Sufi Fragments* is one which owes most to the heritage of Western classical music, but it is built on a harmonic vocabulary which is in many — but not all — respects a union of 7 (or 10, depending on how you count them) sonorities of 3, 4 and 5 pitchclasses which occur ubiquitously in PFRJW music, where they are effectively treated as *consonances*. PFRJW idioms are based on a concept of consonance in which the consonant class of chords includes a few of those with *four or five* pitchclasses, in contradistinction to most of the music of the tradition we call The Common Practice of the Western Classical Tradition — created during the 300 or so years from about the mid-17th century to about the mid-20th — consonances are considered to be of only *three*

pitchclasses (major and minor triads), and the four-pitchclass chords which are deployed (dominant 7ths, minor 7ths, half-diminished 7ths etc) are considered dissonances.

The expansion of the list of consonances in PRFJW musics (and in *Sufi Fragments*) beyond the major and minor triads of the Common Practice stems from the addition of the major second as a component interval, as well as thirds (major and minor) and perfect fourths and fifths in all of them. Nevertheless, there are also a few differences between PRFJW practice and that of *Sufi Fragments*. For example: the minor second — quite commonly found within harmonies such as the major 7th (chords ubiquitously deployed in, for example, such PFRJW compositions as those of Bacharach), is *not*. This is not just a personal compositional decision, however. It's because *Sufi Fragments*, being grounded in the *Extended Common Practice* (see below and Tymoczko (2011)), and its larger repertoire of consonances (including consonances of 4 and 5 pitchclasses) rather than the *standard* Common Practice and its smaller (3-pitchclass) repertoire of consonances, needs to take a slightly modified approach to making the important distinction between essential and inessential dissonances (see below and Damschroder (2006)).

In *Sufi Fragments* this larger repertoire of consonances is, however, largely treated simply as collections of pitchclasses, and the movement from one to the next determined by voice-leading and by common pitchclasses, not by root progression or other factors which may be important in PFRJW music. Because of this, which note is the root, which is the third, or indeed which is any other component of the chord, is often ambiguous. Hence, the descriptions the composer (GH) deploys are usually the set-theoretic ones, ie descriptions often used for 20th-century *classical* music sonorities, in preference to terminology derived from that used in connection with PFRJW musics themselves.

Other explanatory terms are deployed in a way common in classical music contexts too (*Sufi Fragments* is, after all, a classical piece in essence). For example, the familiar (tonal theory) concept of inversion is about which tone is in the bass. It is also called position (root in the bass = first position, third in the bass = second position, fifth in the bass = third position, etc). When the term *inversion* is used in the present context, however, it usually means inversion of the *component intervals*, not a redistribution of the component pitchclasses in space. Thus the major triad in *any* spatial disposition might be described as "047" (considered abstractly simply as pitchclasses: an interval of 3 semitones above one of 4) and the minor triad in *any* spatial disposition as "037" (considered abstractly simply as pitchclasses: an interval of 4 semitones above one of 3).

Furthermore: in the listing which follows, of 7 consonances (in the above sense) — or 10 if inversions (in the above sense) are counted separately — and which is exemplified in Figure 1, the choice of transposition level on which to begin listing each row of the figure is simply arbitrary. In all of these sonorities, the alphabetical nomenclature remains unchanged for 19-EDO, but all of the set-theoretic integer nomenclatures will be modified as indicated in the following list. So: in a 12-EDO context, set-theoretic integer nomenclature refers to semitonal distances (1/12 of an octave) from a given arbitrary "zero-point". In a 19-EDO one, it refers to distances of 1/19 of an octave from a given arbitrary "zero-point". In passing, we ought to note that the names by which these sonorities are known are rather various in different language-traditions (German, British, American et al) and in different sub-cultures (classical, pop, jazz, etc), so we have thought it best to give a bunch of alphabetical examples and the bland set-theoretic integer-names, along with one or other of the possible names, realising that some readers may be accustomed to a different name in any individual case.

Figure 1, row 1: the Major-minor 7th with suspended 4th: example in alphabetical nomenclature = GCDF, set-theoretic nomenclature (12-EDO) = 0257, set-theoretic nomenclature (19-EDO) = 0-3-8-11.

In rows 2 and 3 of the the figure, the sonorities are inversions of one another (in the above sense).

Figure 1, row 2: the minor triad: example in alphabetical nomenclature = ACE, set-theoretic nomenclature (12-EDO) = 037, set-theoretic nomenclature (19-EDO) = 0-5-11.

Figure 1, row 3: the major triad: example in alphabetical nomenclature = GBD, set-theoretic nomenclature (12-EDO) = 047, set-theoretic nomenclature (19-EDO) = 0-6-11.

Figure 1, row 4: the added sixth: example in alphabetical nomenclature = CEGA, set-theoretic nomenclature (12-EDO) = 0358, set-theoretic nomenclature (19-EDO) = 0-5-8-13.

In rows 5 and 6 of the the figure, the sonorities are inversions of one another (in the above sense).

Figure 1, row 5: the major triad with added second (GABD): example in alphabetical nomenclature = GABD, set-theoretic nomenclature (12-EDO) = 0247, set-theoretic nomenclature (19-EDO) = 0-3-6-11.

Figure 1, row 6: the minor triad with added fourth: example in alphabetical nomenclature = AGFD, set-theoretic nomenclature (12-EDO) = 0247, set-theoretic nomenclature (19-EDO) = 0-3-6-11.

In rows 7 and 8 of the the figure, the sonorities are inversions of one another (in the above sense).

Figure 1, row 7: the dominant seventh or Major-minor 7th: example in alphabetical nomenclature = GBDF, set-theoretic nomenclature (12-EDO) = 0258, set-theoretic nomenclature (19-EDO) = 0-3-8-13.

Figure 1, row 8: the half-diminished seventh: example in alphabetical nomenclature = BDFA, set-theoretic nomenclature (12-EDO) = 0258, set-theoretic nomenclature (19-EDO) = 0-3-8-13.

Figure 1, row 9: the dominant ninth or Major-minor-major 9th: example in alphabetical nomenclature = GBDFA, set-theoretic nomenclature (12-EDO) = 02469, set-theoretic nomenclature (19-EDO) = 0-3-6-9-14.

Figure 1, row 10: the "dominant-seventh-sharp-eleventh" (GBFAC♯): in which the fifth is usually omitted (always omitted in *Sufi Fragments*, making it a 5-pitchclass sonority). Also: noting that the ninth (A) is present in the chord, the description "dominant-ninth-sharp-eleventh" would be possible. Both nomenclatures can be found in the PFRJW literature. Since in *Sufi Fragments* the sonority is treated as a bunch of pitchclasses (5 of the 6 pitches of the whole-tone scale) in any case, the distinction of nomenclatures is minimally relevant to present circumstances. Example in alphabetical nomenclature = GBFAC♯, set-theoretic nomenclature (12-EDO) = 02468, set-theoretic nomenclature (19-EDO) = 0-3-6-9-12.

All of these consonances — 7 or 10 of them, depending on whether inversionally-related sonorities (eg those in rows 2/3, 5/6 and 7/8) are counted separately or together — exclude the minor second.

Because of a combination of

1. progressive tonality,
2. comprehensive and constant circulation of all 12 (or all 19) pitchclasses, and
3. the expanded vocabulary of consonances (compared to that of the Common Practice),

the treatment of dissonance in *Sufi Fragments* is slightly different from that of the Common Practice, and indeed not identical to the PFRJW literature either.

The venerable terms *essential* and *inessential* to categorise dissonance are usually attributed to one of the *loci classici* of Common Practice music-theoretical writing, that of Johann Philipp Kirnberger (1721-1783):

"In the first volume of his magnum opus *Die Kunst des reinen Satzes in der Musik* (1771), Kirnberger asserts that . . . one must distinguish between incidental dissonances which embellish the underlying harmonic formation and the consonances and "essential" (*wesentlich*) dissonances which constitute the harmony." (Damschroder 2006:37)

So: in *Sufi Fragments*, dissonances such as the semitone are normally treated as *inessential*. The implication is that they are resolved within the consonant sonority, and are created by passing and neighbour tones (accented and unaccented) or other embellishing tones. When *essential* dissonances ("stand-alone" sonorities which include the semitone) occur (more rarely) they usually contain 5 or 6 pitchclasses. An informal comparison between the treatment of dissonance in Renaissance polyphony and Baroque figured bass practice could be made for an off-the-cuff impression of the difference. Hence 4-pitchclass sonorities which sometimes occur as stand-alone sonorities in PFRJW music, eg the major seventh chord (CEGB or 0158), are usually part of larger (5- or 6-pitchclass) *essential* dissonances, or constitute *inessential* (passing, neighbour or embellishing) dissonances.

The harmonic vocabulary of the *Sufi Fragments* is based on extensive deployment of these 7 consonances and various transformations (transpositions, inversions) of them. These transformations include transpositions to every degree of the chromatic scale, as well as inversions of each consonance and its transpositions to every available scale-degree, in cases where the process of inversion generates different sets of pitchclasses from any of the transpositions (as with the "added second/added fourth" chord, the dominant/half-diminished 7th and the triad). Figure 1 lists this vocabulary of 120 consonances for the 12-EDO scale.

3: How big is a major second?

In the 19-EDO scale, the same chords and intervals can be replicated (to within scalar variation), except that the interval we identify as a major second is 189 cents rather than 200, the minor third is 316 rather than 300, the major third is 379 rather than 400, the perfect fourth 505 rather than 500, the augmented fourth 568 rather than 600, and the diminished fifth 632 rather than 600, and except that there would be 19 consonances in each of the 10 rows of the consonance-vocabulary chart listed in figure 1, rather than 12, ie 190 consonances altogether. *Sufi Fragments* are miniatures, so only a selection of these 310 consonances (120 or 190) have the scope to appear, but in compositions by the same composer of greater extent and scope, which share more or less the same compositional idiom, all or virtually all of these 120 or 190 consonances often appear somewhere in the course of the piece. A judicious selection of the aforementioned 5- and 6-pitchclass essential dissonances

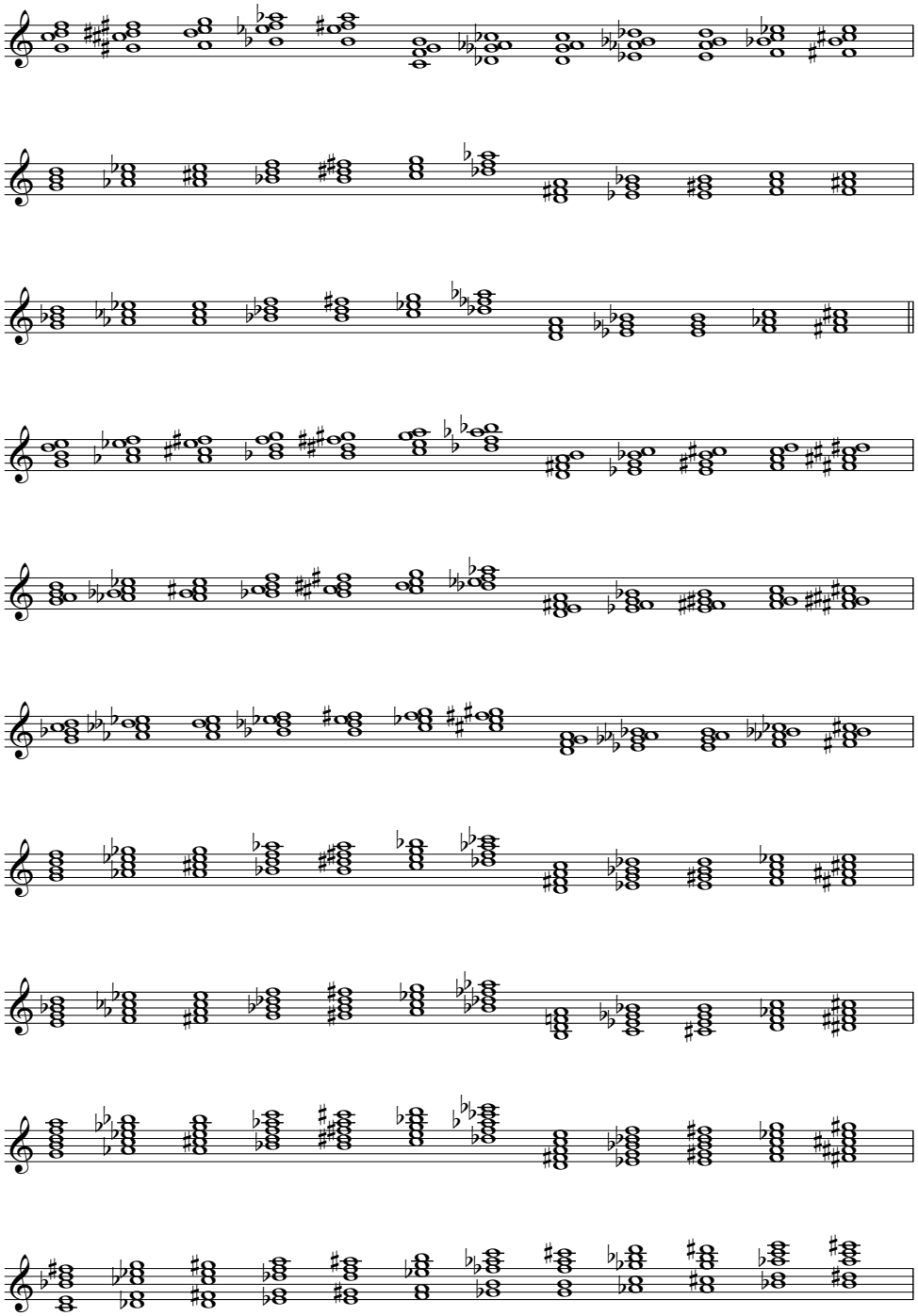

Figure 1: Chords for *Sufi Fragments* by Graham Hair

supplements these 120 or 190 consonances from time to time, but a detailed consideration of these would take us beyond the scope of this essay.

In passing, we might note that, despite the above-mentioned commonalities between 12-EDO and 19-EDO, in the context of *Sufi Fragments*, the mention in the preceding paragraph of the the tuning of the augmented fourth as 568 cents rather than 600, and of the diminished fifth as 632 rather than 600, indicates that the enharmonic equivalences in 12-EDO and 19-EDO are not identical (albeit consideration of that fact, while relevant to some compositional and performance issues, would also take us beyond the scope of this essay).

The underlying scale structure of *Sufi Fragments* is definitively the *chromatic* scale — whether 12-EDO or 19-EDO — rather than an underlying scale of between 5 and 8 notes modified by multiple "modal inflections" (ubiquitous flattening or sharpening of scale-degrees in a context governed essentially by standard Common Practice harmony, with its scale-degree-related functionality): modal inflections which, *in toto*, might well result in the deployment of all 12 or all 19 pitchclasses over the course of a piece (especially one of more than miniature length). By *chromatic scale* in this content, then, we just mean the superset of all the pitchclasses with distinct tunings (ie 12 or 19) deployed: ie essentially the tuning of sets of 12 or 19 keys on a keyboard which, of course, can be subject to an infinite variety of intonational variation by singers, within and between notes.

Nevertheless, a lesser number of pitches is usually in play in any chunk of *Sufi Fragments* of about 3 – 5 measures: what Dmitri Tymoczko (Tymoczko 2011) calls "limited macroharmony". Limited macroharmony covers the pitchclass collections in play at an intermediate level, ie above that of the individual chord and its associated scale (viz at the level of about 3 – 5 measures), but below that of the pitchclass collections deployed overall in the piece or section (the level of 20 – 35 measures, in the case of each of the *Sufi Fragments*). Such limited macroharmony is partly due to the relatively slow-moving harmonic rhythm of *Sufi Fragments*: contrasting with those idioms (eg twelve-tone) where the macroharmony (pitchclass-turnover at the scale of about 3 – 5 measures) is essentially similar to the harmonic vocabulary of the piece as a whole (ie pitchclass turnover at the 20 – 35 measure scale).

To consider one interval, the major second between C and D: an interval which, when deployed harmonically (or indeed melodically) within the context of the 12-EDO scale, is 200 cents in size. When the major second is deployed within the context of 19-EDO scale, the interval is 189 cents. We can report, from personal experience, that the singers have no trouble at all in acclimatising themselves to the interval of the major second whichever of the two scalar contexts is the relevant context at the time. That is not to say, of course, that their intonation approximates 19-EDO. Kopiez (see below) suggested that his trumpet soloist, given several versions of the accompaniment of a piece of music in different temperaments (meantone, Pythagorean, equal temperament, etc) continued to play in something approximating 12-EDO all the time. This practitioner (GH) suggests a slightly different interpretation of Kopiez's results, namely that the trumpeter played a kind of *generic diatonicism/modalism* which would "fit in with" any of these temperaments fairly well, while observing *none* of them very strictly.

Of course, the music examples which Kopiez chose may well have had a bearing on the situation. In pieces whose harmony can be characterised as Common Practice, certain of the 12 pitchclasses (and of the 11 intervalclasses) recur much more often than others. In the 21st-century harmony of *Sufi Fragments*, which conforms fairly well to what Tymoczko characterised as the ECP (the *Extended* Common Practice), this is only partially true. In particular, the frequency of occurence

of the 19 pitchclasses is much more equal than in the case of the "standard" common-practice repertoire. In more extended pieces especially, all the vocabulary of pitchclasses (whether 12 or 19) recur with more or less the same frequency, and the harmony rarely terminates on the same tonic with which it began; something which sometimes appeared in earlier music too (eg Mahler, though instances can be found as early as Schubert), and was characterised as *progressive tonality* . . . or in something rather similar, namely (to use Schoenberg's term) "roving harmony". Following Tymoczko's principle of *limited macroharmony* (". . . the total collection of notes heard over moderate spans of musical time . . . often involving between 5 and 8 notes"), the music of *Sufi Fragments* cycles continuously through a series of *different* diatonic/modal collections, which, over larger spans of time (typically 20 – 35 measures, in contradistinction to limited macroharmony's typical span of 3 – 5 measures) bring about this overall equality of distribution. Thus the singers are forced to accommodate themselves to continually-changing local diatonic/modal environments of between 5 and 8 pitchclasses, but summing to 19 in the overall course of the piece. This is not to suggest that the singers' intonation in *Sufi Fragments* conforms any more strictly to 19-EDO than Kopiez's trumpeter to his different temperaments. The hypothesis is that a series of *generic limited macroharmonies* describes the situation better.

Nevertheless, experience has shown that listeners to 19-EDO and other less common tunings often have a (somewhat vague, but nevertheless palpable) impression that their overall sense of scale, tuning and intonation deviate in some way from the 12-EDO to which they have been long accustomed. Naive listeners sometimes opine that — like such phenomena as traditional Turkish Music or Harry Partch's "Just Intonation" works — the music is "out of tune", while more sophisticated listeners with greater experience of the great diversity of the world's teeming thousands of musical cultures, who realise — implicitly at least — that there is no absolute standard of "correct" tuning and temperament, are quicker to intuit the element of cultural difference responsible for such phenomena.

In each case, although the accompanying music in the digital harmonium part might be either precisely 200 cents or precisely 189 cents, we might want to say that the singers' rendition is, at best, no more than *approximately* 200 or *approximately* 189 in each case. Even to say that, as far as the singers are concerned, the interval of a major second is a kind of average of these values (194.5 ± 5.5 cents) is to disregard the fact that 5.5 cents is really beyond the discriminatory powers of any singer. So we might therefore prefer to suggest that a singer's intonation of the major second, even if the keyboard accompaniment is precisely 12-EDO or precisely 19-EDO, would probably be somewhat less than 189 or somewhat more than 200 cents: that is, 194.5 ± 10 cents (at least!). We reason, from such experiences, but also from other experiences which we shall describe in due course, that — essentially — intervals are *psychological* phenomena and that singers can easily accommodate to a variety of sizes in the appropriate context (albeit not to absolutely any size whatever!), and still recognise that interval as a major second. That context might be cultural (the interval-sizes to which they are accustomed) as well as perceptual (the ability of the human perceptual apparatus to employ categorical means of sensing the size of any interval, and to disregard discrepancies between different representations of the major second — or any given interval — when it seems to the singer to make musical sense to do so).

It is also true, of course, that the many music-theoretical pedagogical treatises which discuss classical Common Practice harmony often begin with a representation of the harmonic series in standard musical notation. For example, here is the representation of the harmonic series which

might come from any one of hundreds of elementary primers:

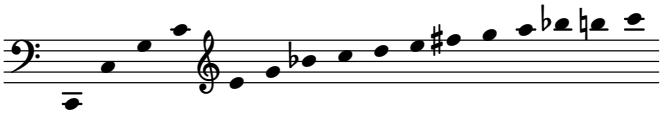

Figure 2: Harmonic Series on C2

In this figure, the series whose fundamental is C2 represents the 7th, 8th, 9th, 10th and 11th partials by the pitchclasses B♭4, C5, D5, E5 and F♯5, so that each of the 4 adjacent intervals between these 5 partials is considered as a major second. Nevertheless, these intervals which we place perceptually in the same category ("major 2nd") are clearly not acoustically the same size. If we calculate them, or measure them empirically, their respective sizes are (approximately) 231, 204, 182 and 165 cents, or, in terms of ratios between frequencies, 8:7, 9:8, 10:9 and 11:10. Harry Partch, for example, employed *all* of these intervals, including the largest (8:7) and the smallest (11:10) in his scales and harmonies. These representations of the "major second" mean extending our interpretation of it enormously, to encompass a range of about 200 ± 30 cents.

We have already noted that the interval of a major second, when deployed within the context of PFRJW music is basically regarded as, and treated as, a consonant interval. This practice has been adopted in *Sufi Fragments*, where what is intended is a form of the "Extended Common Practice", ie encompassing the practice of the Western Classical music tradition for as long as the whole millennium from 1016 to 2016. The point at this stage in the discussion, however, is not that the concept of consonance has shifted somewhat (albeit not totally!) over time, but that — perhaps — the range over which the interval of a major second is considered to vary seems to be as much as ± 30 cents.

The extent of the sense of consonance of any given interval is due to the extent of the (approximate) coincidence of the partials of its component notes Helmholtz (1863, 1998). Thus, if (cf Figure 2) one considers the first 16 items of an harmonic series formed on each of a pair of pitchclasses — for example the major second C and D — 3 of the 16 items (C D and E) are shared to within a close degree of approximation. C provides partials 1, 2, 4 8 and 16 of the series on C and the 7th and 14th partials of the series on D. D provides partials 1, 2, 4, 8 and 16 of the series on D and the 9th partial of the series on C. E provides partials 5 and 10 of the series on C and the 9th partial of the series on D. And so on with F♯, A and B.

Because the items in an harmonic series are related by ratios between the frequencies, this sense of coinciding partials may give rise to the idea that intervals may be *defined* in terms of ratios, in other words that — for example — major seconds defined in terms of the interval between the 8th and 9th partials of an harmonic series or between the 9th and the 10th partials, viz intervals of 204 or 182 cents, are somehow *better* or *more precise* major seconds than major seconds consisting of 189 or 200 cents. But this is not the usual situation with regard to the *actual practice* of musical performance, where, as our experience shows, the intervals of a major second (and indeed other categories of interval) can vary considerably, according to a much greater range of performance considerations.

The importance we sometimes attach to defining the interval between C and D as a ratio is, we suggest, a *by-product* of the approximate alignment of partials, or, in other words, a function of the major second within the context of a certain degree of *consonance*. If one considers ratios in which

numerator and denominator are large enough, there are obviously *several or many* ratios which could be taken as definitions of the distance of a major second, and given *any* distance in cents (eg 200 or 189), one can always find *many* ratios which approximate to it, and the definition therefore needs to be modified to identify the *smallest-integer ratio* (ie in this case 9:8 or 10:9), despite the wide variety of the embodiments of the major second in non-keyboard performance situations, and that we have no difficulty at all in recognising these as "major seconds" as commonly as with any keyboard embodiment.

There is, however, another reason for starting this article with a focus on the interval of a major second, in addition to the fact that the major second is a component interval in the selection of 7 (or 10) four-note consonances deployed in contemporary PFRJW idioms (and in *Sufi Fragments*): namely, that the interval of a major second is the most ubiquitous melodic interval in musics worldwide (Vos and Trost 1989:383-396), so it makes sense to begin by asking questions about the extent and limits of its intonation. The reason for the term "major second" implies indeed that, although in *Sufi Fragments*, occurences of the intervals of a major second happen within a context in which the 12-EDO and 19-EDO chromatic scales are fundamental, many musics worldwide deploy the major second as statistically the most common interval between the notes of their melodies within a context in which scales of between 5 and 8 notes are fundamental. Indeed, the major second is *called* a second because the major second is the most common interval between the adjacent notes in such melodies.

We might now ask whether there are circumstances under which singers — and listeners more generally — are especially attentive to the coincidence of component partials. One that comes to mind is the music of David Hykes's *Harmonic Choir*, which is characterised by extremely slow harmonic rhythm, in which the present first author (GH) can report (personal experience) a certain *frisson* at encountering a unique musical experience in which the co-incidence of partials is such an overwhelmingly prominent feature (Hykes 2010). One might compare this situation to that of a violinist tuning her instrument, where, characteristically, she plays her open strings A4 and E5 over and over for maybe 10-12 seconds, listening carefully until she senses the coinciding partials to the strongest possible extent. If one again takes the first 16 partials of each of the notes A4 and E5, this *scenario* means that 12 of those 32 partials consist of the pitchclasses E and B. In such a *scenario*, the interval between A and E — whose designation as a "perfect fifth" reflects the influence of the scale of between 5 and 8 notes — is likely to approach the ratio 3:2 (702 cents, again $\pm$ a few cents, albeit in this case, "few" probably means quite a few less than 30: possibly even the $\pm$ 5.5 which we dismissed earlier as unlikely in other circumstances). Another comparison, having in mind the history of music theory, might be with the historical device known as the *monochord* (*Wikipedia*: "an ancient musical and scientific laboratory instrument") designed so that a comparison could be made between vibrating strings of different lengths and the pitches of the resulting sounds, and the relationships between length and pitch remarked.

Easley Blackwood uses the term "recognizable" to describe the intervals and scales he investigates in his book about the tuning of intervals, scales and harmonies (Blackwood 2014), where he concludes that even the category "perfect fifth" (a fundamental interval of most tonal systems, and one usually considered, under conventional wisdom, to be the interval to whose precise tuning human beings are most sensitive) is indeed "recognizable" over the range 685 to 720 cents, and he composed a whole series of works for electronic keyboards in which the keyboards were tuned to all the available equal divisions of the octave from 13-EDO to 24-EDO, in which a huge variety of different representations

of many different intervals familiar from other repertoires is embodied.

But the circumstances of the *Harmonic Choir*'s performances, of tuning a violin, and of experimenting with a monochord are all special — extreme — performance situations. They have limited application to the situation in "normal" musical performances, especially those where the moment-to-moment intonation is under the control of the performer, ie in the playing of string, woodwind and brass instruments, and — above all — in singing, rather than in performances by keyboard and keyboard percussion instruments. This is the motivation for our distinction here between *tuning* (applying particularly to keyboards, where the pitches of notes are pre-set before the act of performance) and *intonation* (applying particularly to singers, where the pitches of notes are controlled moment-to-moment by the performers, and to non-keyboard instruments), although there is of course a degree of overlap between the two concepts.

4: What is a musical interval?

Our conclusion is that, in "normal" musical performance situations, it makes more practical sense to treat intervals as *approximate distances on a one-dimensional scale* than as ratios. When keyboards are tuned to the 12-EDO scale, the perfect fifth is tuned to an interval of approximately 700 cents; when tuned to 19-EDO it's approximately 694 cents. Such tuning is a feature not only of our digital harmonium, constructed for use in the *Sufi Fragments*, but is also related to some extent to historical precedent, in 17th-century harpsichords with split keys, such as those now found in the Reid Museum of Musical Instruments in Edinburgh, ie the 7 "white notes" (F C G D A E B), plus (up to) 12 "black notes", selected from the 7 possible sharps and the 7 possible flats: F♯, C♯, G♯, D♯, A♯, E♯, B♯, and F♭, C♭, G♭, D♭, A♭, E♭, B♭. In fact, the historical precedents for 19-EDO tuning were one of the things which suggested the composition of the *Sufi Fragments* in the first place, although, of course, 19-EDO is probably our contemporary approximation to 17th-century precedents with 19 tones per octave which were embodied by those instruments (where, to be sure, the distances between the consecutive notes of the chromatic scale were probably not entirely equal: something like a version of Extended Mean-tone Tuning was possibly used on such instruments).

5: Science and — versus? — Humanities.

This paper came into being as a consequence of some conversations between the two authors over a period of some years about the purposes behind (specifically) the *Conference on Interdisciplinary Musicology (CIM)*, and (more generally) a raft of attitudes which led to the creation of *CIM*. *CIM* was an initiative of (in particular) the second author (RP), back in 2004. He noticed that some questions concerning the way human beings interpret music had been investigated by scientists as well as by scholars in the humanities and social sciences — eg psychology, sociology, anthropology, philosophy, literature, linguistics and related disciplines — and that in addition, practising musicians (both performers and composers) often have deeply rooted beliefs of their own concerning these questions, even though they may not have published any writings or promulgated any theories *per se* on these topics.

In principle, *CIM* requires authors from two different backgrounds to collaborate on the papers they present: science and humanities, science and practice or humanities and practice. Academics tend to say that they have been doing this for many years, but often the claim can be rather thin. We know of authors who have written papers, and then rung up a colleague and asked him/her to add his/her name to it! The result in such cases is not so much a collaboration as an endorsement

of the work of the author by his/her colleague. And sometimes, it must be admitted, later *CIM* organisers have been less than fanatical about insisting on this dual-author qualification. Moreover, as the editor's prefatory comments to this volume of *SMR* point out, there has risen in recent years, a generation of scholars with qualifications in both science and humanities and even in practice as well. Indeed, in recent years, virtually all educational institutions claim to promote inter-disciplinarity via their "mission statements" and similar paraphernalia. But such claims are, in many cases, rather hollow, calling to mind George Orwell's sarcastic comments on another universally-endorsed word, *democracy*:

"In the case of a word like democracy, not only is there no agreed definition, but the attempt to make one is resisted from all sides. It is almost universally felt that when we call a country democratic we are praising it: consequently the defenders of every kind of regime claim that it is a democracy and fear that they might have to stop using the word if it were tied down to any one meaning." (Orwell 1961:337-351)

Another problem is that what counts as evidence in one field may not do so in quite the same way in another. A scientific theory can, in principle, be subjected to controlled experiments, but that is hardly practicable when it comes to a Theory of History, Politics, Art or Society. Nevertheless, members of Society at Large still hold theories of History, Politics, Art and many other topics with scarcely less seriousness that they hold scientific theories of various kinds to be true: at least in the sense that they act in accordance with them. The Academy is still as partitioned into disciplines more or less as it was 50 years ago, even though Faculties are now often amalgamated into larger administrative categories (called "Schools" in the institution of our first author). But perhaps this has as much to do with the corporatisation of the contemporary Academy, and the pulling of it together under tighter financial and managerial control as with the promotion of inter-, multi- and trans-disciplinarity.

Following some conversations of this kind, the two authors published a long article in *JIMS* (The *Journal of Interdisciplinary Music Studies* (Parncutt and Hair 2011) outlining some thoughts on how we might reconsider the age-old questions of consonance and dissonance in music in the contemporary context. We put forward the view that many factors contributed to the impression in the minds of human beings (in at least our Western culture) of a sense of consonance and dissonance: ie that the consonance/dissonance dichotomy is a multi-factorial phenomenon, based on combinations of numerous *additional* dichotomies which are in some degree analogous. This does not, of course, deny that in various musical contexts, different dichotomies may not be accorded primary or secondary importance, and that, in respect of both ontology and epistemology, the consensus (to the extent that there is any) has changed over the centuries.

6: Musical interval as a scientific concept.

In what follows, we lead on from these thoughts, again prompted by the contemporary context, to what is perhaps an even more basic topic: the ontology of intervals. Again, we shall argue for the involvement of many factors and the important role of context. Nevertheless, we shall argue for the view that intervals are primarily distances along a one-dimensional scale, whereas for centuries, since the time of the semi-mythical figure of Pythagoras, and following the work of Helmholtz in the nineteenth century, musicians have tended to conceive of intervals as ratios between frequencies. In part, we shall also argue that the distances involved are inherently approximate, and that consonance and dissonance play a role creating the extent and limits of categories of distance,

and discuss in passing how composers and performers have for centuries "pushed at the boundaries" of these categories.

We shall also argue that there are many reasons why our view is not universally shared in contemporary culture, and that these include the nature of Modernity itself. Our modest suggestion, prompted by experience of *CIM* and *JIMS*, is that a useful way of taking the discipline of music studies forward, would be for agents of research in science, humanities and practice to work in collaboration in reality as well as in propaganda, and we offer here a small, very specific instance of such a project. The first author (GH) is involved in practice (composing, directing performances — mainly of singers — and creating, recording and editing music with technological resources) and writing (mainly about contemporary composers and music theory), all in 2017. The second author (RP) is a scientist who, for 30 years up to the present day, has written many books, chapters and articles and given quite a few conference addresses about the way in which the human mind and body interprets musical sounds and structures.

7: A scientist's story.

Stevens (1946) presented a general theory of psychological scales and measurement that had important implications for psychoacoustics. It also has interesting implications for music theory, and is further evidence against the Pythagorean concept of musical interval. Stevens's concept of "scale" differed from the musical concept. He considered the problem of perceiving points on a scale relative to each other. For example, is a 100-watt light globe twice as bright as a 50-watt light globe? The answer is no, even if the 100-watt globe is producing twice as much light power, because perceived brightness is not proportional to light power. The musical concept of scale refers to something quite different: a collection of tones of nominally fixed pitches, used for making music. Stevens' general psychological discussion of "scale" is nonetheless relevant for a psychological understanding of musical scales.

Stevens asked whether it possible to measure human sensation, and defined measurement in a very general way as "the assignment of numerals to objects or events according to rules" (p. 677). He then defined four kinds of scale: nominal, ordinal, interval and ratio. Musical scales can be of all four types. They are nominal scales (because scale steps have names: do re mi etc), ordinal scales (because scale steps are in a *fixed order*), interval scales (because intervals have the *same size at different places in the scale*), and ratio scales (because two octaves are perceived as *twice as big* as one octave, two semitones as twice as big as one).

Psychological research has not been supportive of the Pythagorean concept of musical intervals as ratios. Experiments have revealed consistent stretching of musical octaves and other larger intervals in performance relative to simple ratios, and compression of smaller intervals. To our knowledge, no researcher has ever discovered a recording of a music performance by voices or instruments with real-time pitch adjustment (such as typical wind or string instruments) that consistently conforms to either Just or Pythagorean intonation.

8: Intervals as psychological categories.

Burns and Ward (1978) presented pairs of tones to musicians and asked them to name the intervals. The musicians were good at this task and seldom made errors. The intervals in the experiment were mistuned by as much as a quartertone, but the musicians reliably quickly named them relative to the twelve categories of 12-EDO. For example, they quickly identified an interval of 350 cents

as either a minor third or a major third, and an interval of 450 cents as either a major third or a perfect fourth. The authors concluded that musical pitch perception is categorical. Each perceptual category has a label corresponding to a chromatic scale step and a width of about one semitone. The empirical evidence on intonation suggests that musical intonation is acceptable if listeners can correctly categorize the tones in the chromatic scale, that is, if pitches lie within those category boundaries. Under good conditions for performing frequencies exactly and perceiving pitches exactly, the category boundaries for intonation become smaller than the category boundaries for interval recognition. One may recognize an interval as a major third if it lies between 350 and 450 cents, but under ideal conditions it may be considered "in tune" if it lies between 380 and 430 cents.

We can imagine a listener to any such music, in any style or genre. The music suddenly stops and she hears a pure tone with randomly selected frequency in the central musical range. In general, she will hear this tone relative to the prevailing diatonic and chromatic scales, as one of the chromatic steps (the closest) or as a natural or shifted diatonic step. Unless she has absolute pitch, or good relative pitch plus knowledge of the key in which the music was playing, she will not say "aha, that was a D♭" or "aha, that was the fourth scale degree in A♭ major", but she will hear the tone *as if* it were one of those things. If the tone had been tuned differently, but was still within a quartertone of a D♭, she would hear it as the same tone and may not even be aware of the mistuning. She would assume that the musicians had intended to play D♭, even if the tone was clearly out of tune.

This thought experiment also explains why pitch is perceived categorically. In most music, mistunings of a quartertone or even a semitone are commonplace. The ear must be able to tolerate mistunings of this order, otherwise musical appreciation would be impossible. Mistunings of partials within harmonic complex tones are similarly commonplace; a familiar example is the stretched harmonic series in every piano tone (Martin and Ward 1961).

That can explain why partials within a harmonic complex tone can be mistuned by a quartertone or even a semitone without being separately noticed (Moore et al. 1986). These findings imply that the tuning of partials within complex tones is also perceived categorically: individual audible partials are perceived as either belonging or not belonging to a given harmonic complex tone. The empirical literature on the categorical perception of musical pitch implies that intervals do not sound as far out of tune as they really are. We are remarkably good at ignoring mistuning in performance and guessing the pitch a performer is aiming for, as if our perception were shifted toward the intended pitch of each tone (*zurecht hören*: Bruhn (1994), Fricke (1988) and Kurth (1931)). It cannot be otherwise if the results of studies such as Devaney et al. (2012), Kopiez (2003) and Rakowski (1990) are correct. Categorical perception is like a filter that assigns out-of-tune intervals to familiar categories, as if they were in tune. This is a saving grace for all amateur choirs and orchestras, whose intonation is regularly out by a quartertone or even a semitone.

Categorical perception of musical pitch contradicts Pythagorean theory, in which musical intervals are held to have single, exact sizes. The concept of categorical perception allows us to define a musical interval in a quite different way. Each musical interval label corresponds to a continuous range of interval sizes. Any interval in the range 350 to 450 cents may be perceived as a major third. Plus or minus a quartertone might seem like a very big range of uncertainty, but that is essentially what the experiment of Burns and Ward and others established (eg Halpern and Zatorre (1979), Siegel and Siegel (1977)). We are aware of no good empirical studies of intonation in performances of typical music (rather than music in which tuning is most exact) by choirs, string quartets and so on. Such an empirical study would carefully address the measurement issues addressed above

and apply appropriate statistical methods to the evaluation of large numbers of measurements from diverse musical contexts.

So we may ask why modern Pythagoreans cling to beliefs about ratios in spite of the overwhelming empirical evidence to the contrary — for example, the evidence that intonation varies widely in music performance and often deviates systematically from ratios. Several possible reasons can be identified. First, we tend to idealize music because of the wonderful emotions it evokes, seeking a beautiful theory to account for a beautiful phenomenon. We associate positive emotions with tones that sound exactly in tune, and hence harmonious with other tones. Music is a form of culture that shares utopian features with the Platonic and neo-Platonic thought of the ancient and medieval worlds, but it is not the only one.

A second reason why many still cling to Pythagorean beliefs is that they have successfully explained aspects of music for very long historical periods. In the history of science and ideas, we can observe a general reluctance to give up previously successful theories. Like Kuhn's paradigms in the history of science (Kuhn 1962), Pythagoreanism may only be overthrown when its failures become intolerable and the successes of an alternative become compelling, just as classical mechanics was overthrown by quantum mechanics and relativity. Implicitly such failures are embodied in the reaction of the singers to the 12-EDO and 19-EDO examples with which we started this discussion.

As an approximation, however, the Pythagorean approach will always be useful, just as classical mechanics is still useful. From a scientific viewpoint, a paradigm change in the concept of musical interval may already have happened; but some Pythagorean concepts are still considered valid, at least in certain contexts or domains, for example by humanities scholars or musical practitioners. Musical practice is constantly changing in accordance with changes in the economic, sociological, philosophical, historical, political, ideological and other circumstances; humanities disciplines are consequently "devoted less to the advancement of knowledge than to the propagation of moral and intellectual values" (Scruton 2007:3). Thus, the problem may lie in difficulties of interdisciplinary communication and the mutual recognition of knowledge across disciplinary boundaries as recognized for example by Snow (1959).

Third, we tend to exaggerate the "exactness" of the so-called "exact sciences". Physics is often thought of as an "exact science" because it is dominated by mathematical theory. But mathematics is also an excellent tool for dealing with inexactness. One approach is the order-of-magnitude estimate: the size of a quark is less than $O(10^{-18})$ m; the weak nuclear force operates over a range of $O(10^{-14})$ m; the temperature of a supernova expansion is $O(10^{10})$ Kelvin; there are $O(10^{14})$ cells in the human body; and there are $O(10^{80})$ fundamental particles in the universe. Music theory is often considered mathematical and therefore exact, but in fact there is no clear relationship between mathematics and exactitude. Applied mathematics varies along a spectrum from very exact to very approximate. The number ratios that correspond to musical intervals also lie somewhere along that spectrum.

9: Ontology and Epistemology in context(s): historical, philosophical, sociological, political, anthropological ...

Neither of the two foregoing accounts of specific, practical, personal experiences should lead us to lose sight of the fact that many widely-shared beliefs about music are much influenced by factors other than empirical observation — in the form of historical, philosophical, sociological, political and perhaps anthropological ideas — where the issues may not be primarily those of truth, falsehood

and the advancement of knowledge as much as the propagation of aesthetic, ethical, moral, cultural and intellectual values and *mores*: in which circumstance it is difficult to imagine how it would ever be possible to navigate around ideological implications.

That is to say that all of the aforementioned academic disciplines — history, philosophy, sociology, politics, anthropology — carry within them large consignments of conventional wisdoms embodying such ideological implications, which — on the one hand — enable us to negotiate our way through complex forests of competing ideas, but — on the other — constitute an influence which it is difficult to slough off, even when times change and the usefulness of some established conventional wisdoms has withered away and alternative ones are gaining traction.

Musicologist Leonard Meyer (1994: 328) expressed the give and take between empirical observation and ideological values in terms germane to our topic when he wrote that "... realms external to music affect the history of music by changing the beliefs and attitudes ... that establish the values and goals in terms of which composers and performers, patrons and audiences choose particular musical means." (Meyer 1967, 1994).

Richard Taruskin (2010:340) put this thought even more strongly: "... we all — composers, performers and listeners alike — have strongly vested material, moral, intellectual and spiritual interests that we make our music serve."

For example, in the fourth century BCE, philosopher Plato one of the foundational thinkers of Western civilisation, believed that the physical phenomena of everyday objects and human existence were an imperfect realisation of an ideal perfect world lying beyond them, and that human existence should endeavour to attain to that perfect world. A glimpse into that utopia was provided by the world of numbers, underlying every aspect of God's perfect world, including the celebrated *Harmony of the Spheres*, of which humankind's music was an imperfect realisation.

Nicomachus of Gerasa (circa 60 – circa 120CE) was a neo-Pythagorean mathematician of late antiquity, whose *Manual of Harmonics* (a primer or handbook of received thought, rather than a work of original ideas) was one of the earliest influential works of music theory written in the period after Aristoxenus (circa 375 – 335 BCE) but drawing on traditions from the earlier ancients, going right back to Pythagoras (428/427 – 348/347 BCE). In particular, there is a Pythagorean tradition transmitted through such writings as Plato's *Timaeus*: a tradtion which focuses on the mathematical interpretation of nature. As Kahn says: "... the Pythagorean-Platonic tradition of the *Timaeus* has, in a sense, retained its vitality down to our own day..." (Kahn 2001:153).

One numerical notion considered to embody such utopian aspirations imperfectly to represent God's perfect world was the so-called *tetractys*, sometimes called the "mystical tetrad": a geometrical figure of points grouped in rows of 1, 2, 3 and 4 (cf Figure 3) ...

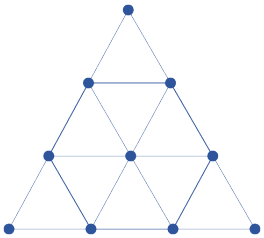

Figure 3: The *tetractys* symbol

. . . a structured enumeration of the first ten integers, and which was thought implicitly to represent the underlying mathematical connectedness of the structures and forms of all things, from the workings of the smallest element in the nature to the largest: a mystical representation of the perfect order of the universe. Dantzig even quotes what is thought to have been an ancient prayer to the *tetractys*, composed by Pythagoreans:

"Bless us, divine number, thou who generated gods and men! O holy, holy Tetractys, thou that containest the root and source of the eternally flowing creation! For the divine number begins with the profound, pure unity until it comes to the holy four; then it begets the mother of all, the all-comprising, all-bounding, the first-born, the never-swerving, the never-tiring holy ten, the keyholder of all." (Dantzig 1930, 2007:42).

To most of us in the twenty-first century, such formulations undoubtedly sound like hocus-pocus, but, perhaps the *tetractys* can be understoood simply as a symbol for the perfection of the universe which numbers were felt to represent. Indeed, the idea that there must be a basic principle underlying the order of the universe is still with us in the form (for example) of hypothetical models of the so-called Grand Unified Theory, unifying the four physical forces of gravity, electro-magnetism and the strong and weak nuclear forces into a single, all-encompassing, coherent theoretical framework of physics that fully explains and links together all physical aspects of the universe.

Some basic and persistent doctrines in the discipline of music-theory can also be traced back to ancient ideas such as those of Plato and Pythagoras: often as transmitted through medieval culture, as, for example, via doctrines found in the writings of Boethius (480 – 524 CE), one of the music-theory's founding fathers, who is usually considered to be a neo-Platonist.

Leonard Meyer (1967, 1994), writing nearly a millennium and a half after Boethius, was nevertheless able to advance a reason why ancient quasi-mystical numerological formulations have sometimes persisted in music-theoretical thinking even into the late twentieth century: namely that utopian ideologies have continued to hold sway over contemporary thought about many topics, including music-theoretical ones, even in the post-Enlightenment era, albeit for very different reasons, and with very different results than in ancient and medieval times, indeed in pre-Enlightenment times generally.

"In the early millennia of Western culture — say through the Middle Ages the present was judged in relation to a golden past: an age of uncorrupted Eden, of peerless gods and heroes, or of incomparable civilization . . . And that exemplary order was the authorative source of beliefs and knowledge, values and goals . . . A great revolution in man's conception of the world took place when, between the fifteenth and seventeenth century, the golden age moved from the past to the future . . . Nurtured on the successes of science and technology, the Idea of Progress became an article of faith in the eighteenth and nineteenth centuries . . . The more change, the sooner utopia, whether a perfect democracy, a classless society, or a world without want or war . . . All realms of human activity — the sciences, the arts, philosophy, history and religion — were future-oriented and goal-directed . . . In short . . . one of the most consequential beliefs in any culture is its beliefs about the relationship of the past to the present and especially of the present to the future (Meyer 1967, 1994:328 – 330).

10: The influence of historical, philosophical, sociological, political, anthropological forces on empirical observation

Meyer's comments on the ubiquity of utopian ideas of history (eg that some perfect state of society was, in the ancient world, thought to have been the original state of God's world, a state which

humankind ought to endeavour to approach in society's affairs), serve to remind us that the idea that there could be a perfect society has often been mirrored by the idea that a perfect music — or at least a superior kind of music — can be created, based not merely on the skill of its creators, but on some kind of special insight with power to eclipse, override or transcend such sober, prosaic explications. Indeed, it has been a recurring view of the world of music both ancient and modern for millennia. Nowadays we should probably want to characterise such a view as an ideology. In Pythagorean thought, the world of numbers (eg as embodied in the *tetractys* and its set of pristine small integers) was thought to be one phenomenon through which such perfection — in music, as in nature — might be approached.

Moreover, the latter part of the quote from Meyer implies that utopian ideas of history — so far from being overthrown in the post-Enlightenment world, which so often considers itself more rational than the world of the ancients — have simply metamorphosed, and continue to flourish as vigorously as ever. All that has really changed, in such views of the world, is that the superior state of society's affairs has been transferred from its place at the inception of God's world to an indefinite point in the future, when it would be brought about progressively through human agency. Many streams of thought about the history of music in the 20th century can be comprehended in terms of progress towards such a superior music. Schoenberg's comments to Josef Rufer (that his ideas would ensure the hegemony of German music for 100 years), are only one such example. The music of the *avant-garde* in the second half of the 20th century was in many respects a continuation of that ideology. The very meaning of the term *avant-garde* implies that practitioners thereof who self-identified as pointers to a superior future music, created in the present and worthy of hegemony now and of survival into the generations of the yet-unborn, above that of inconsequential (less progressive) kinds of music.

When Harry Partch created his music, during the second and third quarters of the 20th century, he proposed that the whole of the Western Tradition in the centuries immediately preceding his own, had been "built on a lie" (ie on the rise of tempered tunings, especially the by-then ubiquitous 12-EDO), and he spent a life-time building special instruments which would restore (as he saw it) the ratio basis of intervals.

In Partch's treatment of intervals as ratios, two utopian ideologies — ancient and modern — thus converge. His dream of replacing or at least complementing music "built on a lie" with a superior music built on Truth is only one strain of such a utopian dream. We mention Partch specifically here only because his music revolves specifically around the proposal to restore the ratio basis of intervals, and since, nevertheless, his music offers many other points of interest. However, the music of the 20th century provides a plethora of further examples which its creators imagined as casting aside music built on a lie in favour of a music grounded in Truth. In most cases, the Truth to which such dreams were directed was *historical* rather than *acoustical*: the imagining of a superior music evolving in harmony with the *Zeitgeist*, a music which was, so to speak, "on the right side of history".

Often, such dreams meant taking a critical, even contemptuous, attitude to music considered to be on the wrong side. The Nazi state, for example, denounced music which it considered fell into that category as "degenerate". The Marxist states of Eastern Europe and elsewhere provided financial support to musics they considered to be favourably disposed towards the evolution of a prolatarian utopia and discriminated against musics which they considered not to measure up to such a progressive historical ideology as "formalist". In the West, Theodor Adorno (1903 – 1969), also a Marxist of course, took a view diametrically opposed to that of the Marxist states, albeit,

ironically, on similar grounds (ushering in the true music of the democratic age). In his celebrated book *The Philosophy of Modern Music* (Adorno 2007) — entitled *Philosophie der neuen Musik* when originally published in Tübingen in 1949 — he compared the extent of the embodiment of the "correct view" of history in the work of the two best-known composers of his day, Stravinsky and Schoenberg: allocating blame to the former and praise to the latter on those grounds. After World War II, a generation of then-young figures further developed what they considered to be "music of the future": an approach now institutionalised in the late 20th-century and early 21st-century Academy as a view of musical creation as a species of research activity, and the setting-up under the neo-liberal state of Arts Councils and other institutions to distribute funding preferentially to musics considered to meet this most recent interpretation of superiority grounded in Truth, defined as the correct interpretation of the *Zeitgeist*.

The widespread allegiance to such utopian conceptions of music in the contemporary world has only an indirect connection with the world of ratios, of course, except in the specific case of Partch, but it has been one of the forces behind the maintenance of an intellectual climate enabling the ancient idea of a perfect music to live on into the modern world, and facilitated the perpetuation of the idea that there is some special ideological key to the creation of a perfect or superior music, which allows the Pythagorean idea of intervals as ratios to remain in place as a kind of "ancient truism".

But the proposal here is that unbiased consideration of a combination of evidence from the viewpoints of 21st-century practice, science and humanities scholarship should bring us to an alternative view: that there is *no* perfect or superior tuning, just as there is *no* perfect realisation of the *Zeitgeist*, and *no* factors which can eclipse, override, or transcend such universal ancient wisdom as that which defines music as a craft bringing together the deposition of skill, the attainment of beauty and the imparting of pleasure: things to which there is no special ideological key, and which have been the primary purposes of music all along, whatever the sense of the transcendental which responses to them trigger in communities of listeners.

Nevertheless, the idea that such factors are surpassed by the achievement of superior ones such as the attainment of sublimity and higher truth, often through transcendental abstraction, has been a ubiquitous trope of the post-Enlightenment era since the days of Ernst Theodor Amadeus Hoffmann (commonly E T A Hoffmann, 1776 – 1822) and Edouard Hanslick (1825 – 1904) in the 19th century, and has enabled the Culture Industry under Modernity to collude in the denigration of the aforementioned universal ancient wisdom with respect to music, skill, beauty and pleasure as a pre-occupation of inconsequential musics or of the inconsequential intellectual orders of society.

Finally, we may note that, even in the ancient world, there were intellectual dissenters from the Pythagorean outlook, notably Aristoxenus. Nevertheless, the traditions of Western music theory, from late antiquity and medieval through to modern times, transmitted through such dominating father-figures as Boethius have continued in a deeply Pythagorean vein: perhaps one of the reasons why such discoveries as those of Helmholtz are often interpreted in Pythagorean terms. However, the view expressed here is that essentially Aristoxenus was right all along, and that it is now appropriate to give him his due.

Perhaps this is especially the case because utopian ideas of society have been subject to severe challenge since the late 20th century, with the demise of the Marxist states and the widespread scepticism about current neo-liberal ideology (with spreading populism as a common symptom, albeit not necessarily a solution). Moreover, in music, the traditions stemming from such post-

Enlightenment figures as Hegel and Hanslick in the 19th-century and Adorno, Schoenberg and the "post-Schoenberg" generations in the 20th, may finally be fizzling out with such events as the death of Pierre Boulez in 2016, despite the fact that Boulez's music, like Partch's, offers many other points of interest.

The persistence of the construal of intervals as precise mathematical ratios exemplifies a small, specific example of a much more generic utopianism in which may be perceived one of the most widespread illusions of Modernity, the so-called *Nirvana Fallacy*: a ubiquitous 20th-century form of overreaching which blighted many different aspects of 20th-century life, and which denotes elevating dreams of perfect, imagined, unrealistic, idealized solutions, in the place of achievable real things: a refusal to let how things *actually are* stand in the way of how things *really ought to be* (Meyer 1967, 1994).

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